ANALYTICAL TOOLS FOR DESIGN OF FLEXIBLE PAVEMENTS

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Abstract: This paper falls into two parts. In the first, the widely used analytical-empirical method of pavement design and evaluation is discussed and in the second two simulation models are presented. There are important differences between the assumptions on which the theoretical models are based, and the reality of pavement materials and structures, and these differences are important both for the determination of input values (elastic parameters) and for the calculation of pavement response. Linear elastic theory often results in incorrect moduli, when used for backcalculation of layer moduli from deflection testing, and in questionable stresses and strains, when used for forward calculation. Including non-linear materials characteristics may improve the theoretical model, but no theoretical model has yet been conclusively verified with experimental data. The empirical relationships used to predict pavement deterioration from critical stresses or strains, are equally problematic.

In the second part of the paper two theoretical models are presented. The first model simulates the deterioration of a section of pavement over time using an incremental-recursive process. The model is stochastic and considers the spatial variation of pavement materials, layer thickness and traffic loads as well as seasonal variations. The second model deals with the forces on, and the displacements of, the individual grains in a particulate medium. This model does not rely on empirical relationships to predict permanent deformation or failure, but models this in the same process as forces and displacements. The input to this model includes the grain size distribution, the shape of the grains and the degree of compaction, parameters that are similar to those used for specification of materials and quality control.

It is concluded that an interaction between development of theoretical models and experimental verification is needed to improve the understanding and predictability of the complex process of pavement deterioration.

1. Introduction, the Analytical-Empirical Method

Professor S.F. Brown in his keynote address to the 8th International conference on Asphalt Pavements in Seattle in 1997 (Brown, 1997) gave an excellent description of the importance of the “Ann Arbor” conferences for the transition of structural design of asphalt pavements away from purely empirical methods toward a more analytical (or mechanistic) approach, similar to the approaches used for other engineering structures.

The Analytical-Empirical (or Mechanistic-Empirical) approach to design of flexible pavements has two steps, as implied by the name. In the first step the critical stresses or strains (the response) in the individual pavement layers are calculated using an analytical model, and in the second step they are compared to permissible stresses or strains. In the more sophisticated versions, the critical stresses or strains are used for determining the rate of deterioration (the performance).

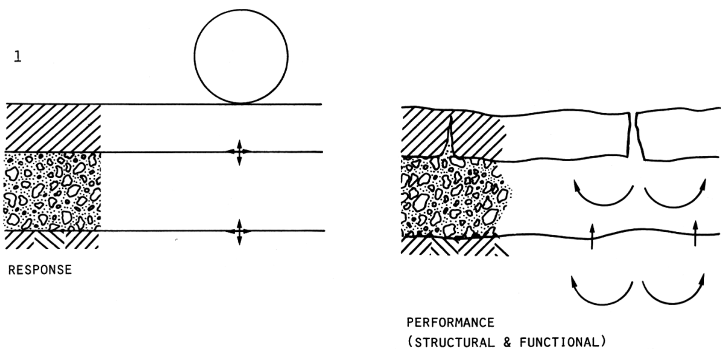

Figure 1
Pavement response (analytical) and performance (empirical)

There is not a strict limit between the analytical and the empirical part of the method. If an equivalent standard axle is used with a mean annual pavement condition, the effects of real traffic loading and seasonal variations, will be included in the empirical part. The tendency is towards extending the analytical part and pushing back the empirical.

For all existing design methods, the analytical tool used to calculate the critical stresses or strains is some version of the theory of elasticity. This may be done for different loads and environmental conditions. The damage caused by the stresses or strains is, almost exclusively, determined from empirical relations.

Almost all of the present analytical methods are derived from continuum (or solid) mechanics and are based on the following assumptions:

- Static equilibrium
- Compatibility (or continuity)
- Hooke's law

Based on these assumptions a fourth-order differential equation can be established and solved (normally by numerical methods) when the boundary conditions are known.

None of the above assumptions are strictly correct for flexible pavements.

- The loads are mostly dynamic, not static
- The materials are not solid, many materials are granular
- Deformations are not purely elastic but also plastic, viscous and viscoelastic, and the strains (or strain rates) are mostly nonlinear functions of the stress condition. Most materials are inhomogeneous and some are anisotropic.

Some of the assumptions made with respect to the boundary conditions, such as layers of infinite horizontal extent or uniformly distributed circular loads, are also incorrect.

To overcome some of these differences between the assumptions and reality, more sophisticated models have been developed. Some of the computer programs based on Layered Elastic Theory (LET) may include visco-elastic or anisotropic materials. With the Finite Element Method (FEM) it is also possible to consider nonlinear characteristics, dynamic loads and/or different yield criteria, but the materials are still, basically, treated as solids. The Distinct Element Method (DEM) is promising for modelling granular materials, but even with DEM it is necessary to simplify with respect to reality.

Even though the assumptions on which a model are based are simplifications of reality, the model may still be useful if the stresses and strains it produces are reasonably close to stresses and strains in real pavements. The only way to find out whether a particular model is useful or not is by comparing the response predicted by the model to the response measured in actual pavement structures.

It is tempting to believe that a more complex model will produce better results than a simple model, but that is not necessarily the case. If the results from the simple model are as good (or better) than those from the more complex model, the simpler model is to be preferred, at least for routine purposes.

2. Determining Elastic Parameters

All analytical models require input of the elastic parameters, as a minimum Young's modulus (E) and Poisson's ratio (n) for each layer. These values may be determined from laboratory tests on the materials, or derived through an inverse analysis of the response measured on *in situ* pavements. In most cases Poisson's ratio is estimated rather than measured.

Triaxial tests may be used on all types of material, but it is not a simple test to conduct. On bitumen or cement bound materials simpler tests such as bending tests or indirect tension tests may be used to derive the modulus. Wave propagation tests or deflection tests are the most commonly used in situ tests, with backcalculation from Falling Weight Deflectometer (FWD) tests presently being the most popular.

The modulus of an unbound material is highly dependent upon the stress condition. A cohesionless granular material, obviously, does not have a modulus as a materials characteristic but only as a function of the stress condition. For analytical design (or evaluation) of pavements, however, it is often the (apparent) nonlinearity of the subgrade that is of most importance.

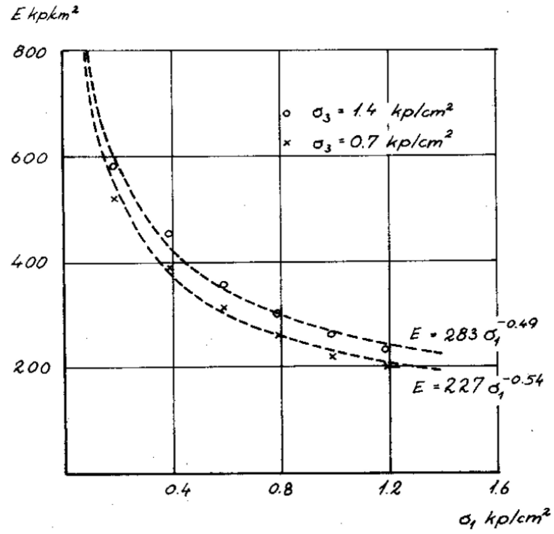

Figure 2
Results of triaxial tests on Keuper Marl
(Brown & Broderick, 1973)

Figure 2 shows the typical variation of modulus with deviator stress for a cohesive soil (clay) (1 kp/cm² @ 0.1 MPa). The experimental results may be fitted with an equation of the form:

$$E = C \times \left(\frac{\sigma_1}{p} \right)^n$$

where: E is the modulus,
σ_1 is the major principal stress,
p is a reference stress, often atmospheric pressure is used, and
C and n are constants (n is negative)

If a load causes a maximum stress of 120 kPa (1.2 kp/cm²) at the top of a subgrade of the material shown in Figure 2, the modulus will be approximately 20 MPa. Deeper in the subgrade or at a larger radial distance from the load, the modulus may be 60 MPa or three times as much.

This type of nonlinearity will have a pronounced influence on the strain at the top of the subgrade, which is often used as an important design parameter, and on the shape of the deflection basin. Fortunately it has very little influence on the stress distribution. For a semi-infinite half-space, the stress distribution in a material with this type of nonlinearity is almost identical to the stress distribution in a linear elastic material (Ullidtz, 1974). In a linear elastic material the modulus has no influence on the stress distribution and the same is, apparently, the case for a gradually changing modulus.

When backcalculating pavement layer moduli from FWD deflections, any nonlinearity of the subgrade must be considered. Backcalulation is an inverse analysis where the layer moduli are estimated, the deflections calculated and compared to the measured deflections, and the moduli are modified until calculated deflections are close to the measured deflections.

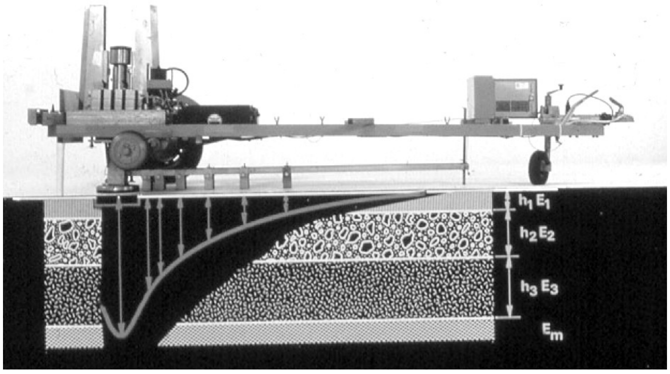

Figure 3
Falling Weight Deflectometer

The outer deflections will depend on the subgrade only, when the distance from the load is greater than the equivalent thickness of the pavement. The subgrade modulus at this distance may be several times higher than the subgrade modulus at the centreline of the load. If the subgrade is treated as a linear elastic material, the modulus must correspond to this high value, otherwise the outer deflections will never agree with the measured values. The subgrade modulus at the centre of the load will typically be overestimated by a factor of 2 to 3. Some pavement design procedures recommend that the subgrade moduli determined from FWD tests, based on linear elastic theory, be divided by a factor of 2 to 3. It would be more appropriate to consider the nonlinearity.

The increase in modulus with distance from the load may be caused by other phenomena, such as a gradually increasing modulus with depth (overburden pressure), the impulse load of the FWD or possibly "stress concentration" (Frölich, 1934), but it appears that treating these effects as an apparent "non-linearity" results in "reasonable" layer moduli and a "reasonably" good agreement between measured and calculated response. An apparent increase of the subgrade modulus with distance may also be caused by a rigid layer at shallow depth. If this is the case, the depth to the rigid layer may be determined from the deflections (Ullidtz, 1998) and considered in the inverse analysis. Sometimes, however, even a rigid layer is better treated as an apparent non-linearity.

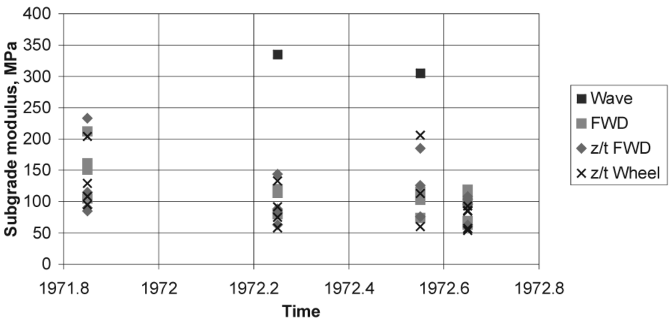

Figure 4
Example of subgrade moduli (till)

Examples of moduli determined by different means, on a clayey sand subgrade (SC, A-4, till, 14% clay), are shown in Figure 4 (Ullidtz, 1973). The moduli were determined at four different test sections during varying climatic conditions. During construction the moisture content varied from 11.7% to 14.2%, corresponding to CBR values of 8% to 3%. Triaxial tests gave moduli from 70 MPa to 180 MPa at a deviator stress of 40 kPa. The test sections were instrumented and the vertical stress (z) and vertical strain (t) were measured at the top of the subgrade. The moduli were backcalculated from FWD deflections (legend FWD) assuming a nonlinear subgrade and were adjusted to the measured stress level, which varied from less than 20 kPa to more than 100 kPa. Values with the legend "z/t" were calculated from the ratio of measured stress over measured strain, under the FWD or a rolling wheel load. On two occasions the moduli were determined from wave propagation tests (Wave).

Although the scatter is considerable (partly because of differences between the four test sections) the agreement between moduli determined by the different methods is reasonably good, with the exception of the wave propagation method, where the modulus corresponds to a much lower stress level.

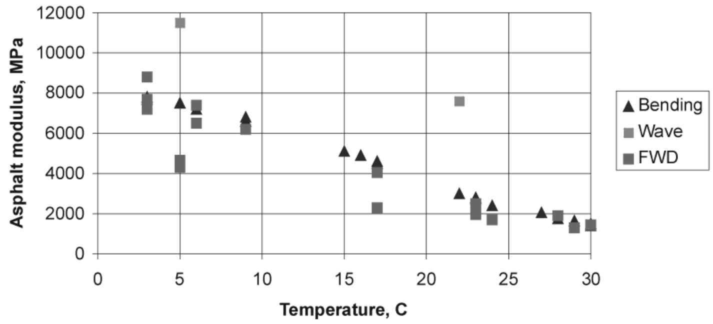

Figure 5
Example of asphalt moduli.

Asphalt moduli from the same four test sections are shown in Figure 5, as a function of temperature. The moduli were determined from three-point bending tests (Bending), from FWD backcalculation (FWD) and from wave propagation (Wave). Again wave propagation results in high values whereas the agreement between moduli determined in the laboratory and through backcalculation is reasonably good (with some outliers).

3. Verifying Analytical Tools

A good agreement between moduli determined in the laboratory and derived from an inverse analysis of measured pavement deflection is a promising sign, but it is not a guarantee that stresses or strains calculated using the analytical method will also agree with measured stresses or strains.

Measuring stresses and strains in pavement structures is far from trivial. There are problems with both the reliability and with the durability of the gauges. Installing a pressure gauge in a material changes the stress distribution in the material, and the environment in a pavement structure is very harsh on the instruments. The last few decades have, nevertheless, seen a considerable development in instrumentation.

In order to evaluate the validity of available response models a research project was carried out under the European Union's 4th framework program. The project was a spin-off of the COST 333 action "Development of New Bituminous Pavement Design Method" (COST 333, 1999), and had the title "Advanced Models for Analytical Design of European Pavement Structures" (AMADEUS). The final report of this project can be downloaded from www.lnec.pt/Amadeus.

During Amadeus 15 different response models were evaluated by 17 European research laboratories and universities. All of the models were compared using theoretical pavement structures, and 8 models were evaluated against measured response from 3 full scale pavement testing facilities (CEDEX in Spain DTU in Denmark and LAVOC in Switzerland). The main tendencies are shown in Table 1.

Table 1
Comparison of response models to measured response

MODEL	TEAM	CEDEX				DTU			LAVOC		
		ε_x	ε_z	σ_z	**d**	ε_x	ε_z	σ_z	ε_x	ε_z	**d**
BISAR	1	⇩	○	↓	⇳	⇩	↓	↓	⬄	↓	⇧
CAPA3D	1	⇩	○	↓	⇧	⇩	↓	↓	⬄	↓	⬄
CIRCLY	1	⇩	○	↓	⇳	⇩	↓	↓	⬄	↓	⇧
	4	⬄	○	↓	⬄	⇩	↓	↓	⬄	↓	⇧
KENLAYER	2	○	○	↓	⬄	○	↓	↓	○	↓	⇳
	3	○	○	○	⬄	○	↓	↓	○	↓	⇩
	4	⬄(1)	○	↓	⬄	⬄	↓	⬄	⬄	↓	⬄(2)
MICHPAVE	3	○	○	○	○	○	○	○	⬄	↓	⬄
NOAH	2	⇳	○	↓	⬄	⬄	—	—	⬄	↓	⇧
SYSTUS	2	○	○	○	○	○	○	○	○	○	○
VEROAD	2	○	○	○	○	○	○	○	○	○	○

The key to the table is the following:

ε_x horizontal strain at bottom of asphalt
ε_z vertical strain in the subgrade
σ_z vertical stress in the subgrade
d deflection at the surface
⇔ Predicted response close to measured response
⇕ Predicted response span the measured response
⇩⇧ Under- or overestimation of response
↓↑ Large under- or overestimation of response

One of the most important conclusions of this project was that the vertical strain at the top of the subgrade tends to be grossly underestimated by the response models, typically by a factor of 2.

The Method of Equivalent Thicknesses (MET) (Ullidtz, 1998) was not included in Amadeus. MET is not an "advanced model" but a very simple method where a layered structure is transformed to a semi-infinite half-space using Odemark's transformation. Stresses, strains and displacements can then be calculated at any point using Boussinesq' equations. One important advantage is that a nonlinear subgrade can easily be incorporated. Even with a nonlinear subgrade all calculations can be done in a spreadsheet.

Because of the disappointing results with the advanced models, MET was used with some of the data. In general the agreement was found to be quite good. An example of measured vertical strain at the top of the subgrade is shown in Figure 6, as a function of the horizontal distance from the load centre. The strain was measured with 4 gauges, with peak values between 1200 μstrain and 1600 μstrain. The strain calculated with elastic layer theory (ELT) had a peak value of 500 μstrain (or 35% of the mean measured peak value), with the finite element method (FEM) the peak strain was 800 μstrain (57%) and with MET 1800 μstrain (129%).

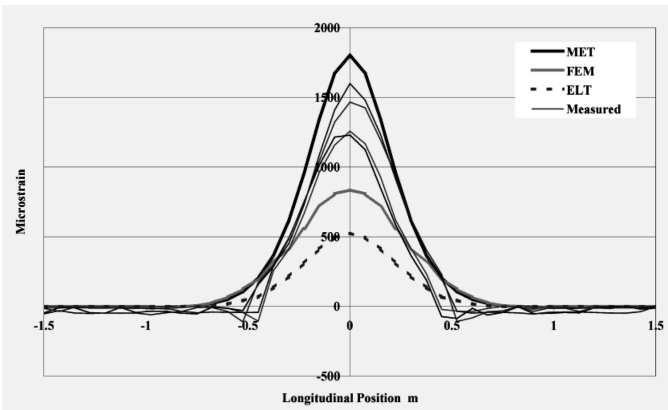

Figure 6
Measured and calculated subgrade strain

A number of other studies, like the International Subgrade Performance Study (Macdonald & Baltzer, 1997, Zhang et al., 1998), have also shown MET (with a nonlinear subgrade) to produce reasonably correct results. It is possible that Odemark's transformation of a layered system is a better approximation to the reality of granular materials than the mathematically exact solution of the 4^{th} order differential equation for layered solid materials.

4. Mechanisms of Deterioration

Once a reliable analytical model is available, the critical stresses and strains may be calculated under different loads and for different environmental conditions. This raises a number of questions:

- How many loads must be considered? Is a conversion of all loads to a standard axle satisfactory, or must the full load spectrum be used?
- What level of detail is needed to describe a load? Is a standard wheel with the load uniformly distributed over one or two circular areas satisfactory, or is detailed knowledge of the distribution of normal and

shear stresses at the tyre-pavement interface required? Should the lateral distribution of the loads be considered? Should the effect of speed on visco-elastic properties or on dynamic loads be included? What about the type of suspension system?

- How do the environmental conditions affect the elastic parameters of the materials? Should critical stresses and strains be calculated for all seasonal conditions (and all loads) or can mean annual moduli be determined from temperature or moisture variations? Must diurnal changes in temperature gradients be included? Should temperature stresses be added to stresses from the wheel loads? and how? How do the environmental conditions influence the strength of the materials?
- Does cracking of bitumen or cement bound materials originate at the bottom of the layer and propagate towards the top? or is it (sometimes) top-down cracking? Can the propagation of cracking be described using fracture mechanics, or is it a more diffuse process which lends itself better to a continuum damage approach? Would a soil mechanics model based on normal and shear stresses be more suitable for asphalt? and how should it relate to fatigue or healing?
- Most design procedures rely on a single relationship between the vertical strain at the top of the subgrade and the number of load applications to predict rutting or roughness. Should different relationships be used for different types of soil, at different moisture contents or degree of compaction? Can similar relationships be used for granular base or subbase materials? Would a soil mechanics model be more appropriate for predicting permanent deformation? How is roughness influenced by spatial variability in materials and layers?

The answers to most of these questions must be found through experimental studies, through Long Term Pavement Performance studies, through Accelerated Pavement Testing and through laboratory testing of pavement materials. But computer simulations may also contribute to the understanding of pavements and pavement materials. In the concluding discussion of the keynote address at the 8th International Conference on Asphalt Pavements Professor Brown states (1997) as future challenge number 1:

"Capitalise on the opportunities for theoretical modelling made possible by innovative ideas and powerful computers."

The following chapters describe two simulation programs, at either end of the spectrum. The first tries to predict the performance of a section of road over an extended period of time and the second is concerned with the movements of the individual grains in a particulate material.

5. Computer Simulation of a Pavement Section

Pavement roughness is an important deterioration parameter. It accounts for roughly 85% of the reduction in PSI (Present Serviceability Index) and the Vehicle Operating Costs (VOCs) calculated in the World Bank's Highway Design and Maintenance Standards model (HDM) are solely a function of roughness. Roughness is the result of variations along the length of a road section, variations in layer thickness, moduli, bitumen content, dynamic loading etc. The increase in roughness over time may be modelled through a computer simulation of a pavement section. This is done in the Mathematical Model Of Pavement Performance (MMOPP) (Ullidtz 1978, 1979 and 1998, Larsen 1986)

The first step in the simulation is to "construct" a length of pavement on the computer. The length is composed of short pieces of road, each 300 mm long corresponding approximately to the imprint of a truck tyre, and the layer thickness, the elastic stiffness, the plastic parameters and the strength parameters are varied from piece to piece. The pattern of variation is quite important to the future roughness, as explained in more details below.

Pavement parameters not only vary along the length of a pavement section, but they also vary over time, as a function of seasonal changes (temperature, frost-thaw, moisture content), of gradual structural deterioration and, sometimes, as a result of ageing. To simulate the gradual deterioration over time, MMOPP makes use of an incremental-recursive procedure, where the output from one time increment (one season) is used, recursively, as input for the next time increment.

Vehicle loads have a static and a dynamic component. The dynamic part will depend on the present roughness of the pavement as well as on the wheel type, suspension system, mass and speed of the vehicle. To simulate the pavement performance the dynamic loads are calculated at each short piece of road and for each vehicle. For each time increment the damage caused by the loads is calculated in terms of reduction of elastic stiffness (in bound materials) and increase in permanent deformation of each pavement layer.

For a given pavement section MMOPP will predict the performance in terms of change in roughness (PSI or IRI), the average permanent deformation (Rut Depth) and the decrease in layer moduli (Cracking) over time, as a function of climate and traffic loading. Because the simulation is based on a stochastic process a different performance will result if the simulation is repeated, just as the performance of two apparently identical sections of road will be different. By repeating the simulation a sufficient number of times the reliability of the design can be evaluated.

5.1 Spatial Variation of Pavement Parameters

For most pavement parameters the value at a specific point (300 mm piece) will depend on the values at the neighbouring points. If, for example, the surface elevation is measured at points along the length of the pavement and the value at point i is plotted against the value at point i-1, then there will be a certain correlation between the two sets of values. This autocorrelation will depend on the distance between the points. With a very short distance the autocorrelation will be close to 1 and it will be decreasing with increasing distance. For very long distances between the points the variation will be random.

Figure 7 shows a longitudinal profile measured for each 0.3 m, and Figure 8 shows a plot of the elevation at point i versus the elevation at point i-1. A regression analysis results in an R^2 of 0.8825, or a correlation coefficient of 0.94. With twice the distance between the points (i versus i-2) the correlation coefficient reduces to 0.80.

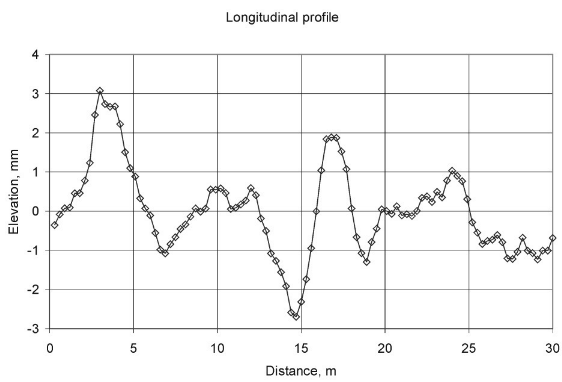

Fig. 7
Surface elevation measured for each 0.3 m

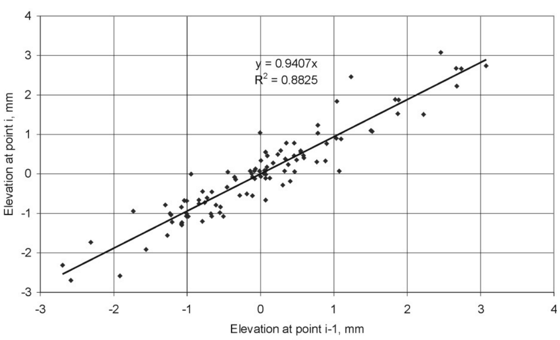

Fig. 8
Elevation at point i plotted against the elevation at point i-1

To generate parameters with a certain autocorrelation function, a second order autoregressive process may be used. In this process the value at point i is obtained from the values at points i-1 and i-2 from:

$$x_i = \varphi_1 \times x_{i-1} + \varphi_2 \times x_{i-2} + a,$$

$$\varphi_1 = \frac{\rho_1 \times (1-\rho_2)}{1-\rho_1^2},$$

$$\varphi_2 = \frac{\rho_2 - \rho_1^2}{1-\rho_1^2}$$

ρ_1 is the autocorrelation coefficient for 300 mm,
ρ_2 is the autocorrelation coefficient for 600 mm, and
a is a normally distributed random variable with a mean value of 0 and a variance:

$$\sigma_a^2 = \sigma_x^2 \times (1 - \rho_1\varphi_1 - \rho_2\varphi_2)$$

where σ_x^2 is the variance of the parameter x.

If the longitudinal profile is generated using a mean value of 0 and a standard deviation of 1 mm, PSI values between 2.4 and 4.0 may be obtained, depending on the autocorrelation coefficients. For the longitudinal profile it is simple to obtain the autocorrelation coefficients, but for other parameters little information is available. It should also be noted that certain parameters, like moduli, do not follow a normal distribution, but are closer to a log normal distribution.

5.2 Seasonal Variations

Both climatic and environmental factors influence the performance of a pavement. In MMOPP the variation of layer moduli with season is described through seasonal factors and for asphalt materials the damage rate is determined as a function of the temperature of the asphalt layer.

Other effects such as temperature gradients, ageing of bitumen or winter salting are not yet included, due to the problems of quantifying these effects. Nor has low temperature cracking been included.

5.3 Dynamic Loads

A simple quarter car model is used for calculating the dynamic component of the wheel loads, as shown in Figure 9. For each vehicle considered in the simulation the masses (M), spring constants (K) and damping coefficients (C) should be given. The lower system corresponds to the axle and the wheel, and the upper system to the suspension. The wheel may be a dual or a single wheel. The tyre pressure is input and, for a dual wheel, also the distance between the wheels.

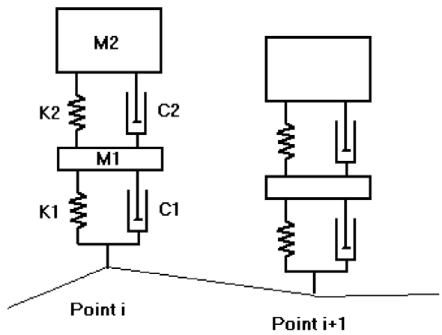

Fig. 9
Mechanical analogue of a quarter vehicle

An example of the variation of the load on the pavement surface with the length of the road is shown in Figure 10, for the load at the beginning of a simulation (PSI about 4) and at the end (PSI about 2). On the smooth road the dynamic load is less than 10% of the static value and on the rough road about 20%, The vibration at a frequency of approximately 2 Hz corresponds to the body bounce of the vehicle, and that at 8 Hz to the axle hop.

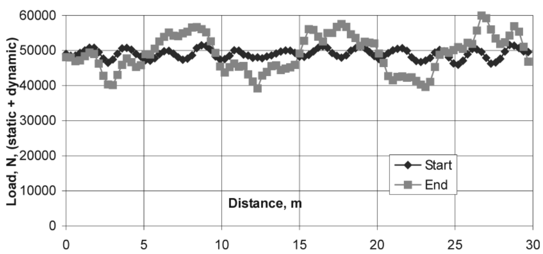

Fig. 10
Variation of load (static + dynamic) on smooth (Start) and rough (End) road

5.4 Continuum Damage Mechanics

Cracking of asphalt and other bound materials may be described as a process consisting of three phases. In phase one diffuse microcracking is formed in the material. In the second phase some microcracks propagate to form macrocracks and, finally, in phase three the macrocracking propagates until fracture.

In fracture mechanics much effort has been devoted to the prediction of the propagation of macrocracking(e.g.

using Paris' law), much less to the emergence of microcracking in phase one and two although according to Kim (1990) most of the failure time is consumed before the crack grows appreciably.

In continuum damage mechanics (Kachanov, 1986) cracking originates as accumulation and growth of microvoids and microcracks. For uniaxial tensile stress a microcrack will cause a reduction of the "active" cross sectional area, from A_o to A. The stress must be transmitted through the remaining intact part of the area. The "damage", w, is defined as the relative area lost:

$$\omega = \frac{A_o - A}{A_o}$$

which (fortunately) is equal to the relative decrease of modulus $(E_o\text{-}E)/E_o$.

In MMOPP the damage rate is assumed to be a function of the tensile strain at the bottom of a bound layer. The damage rate also depends on the type of material and may be a function of temperature, bitumen content and a crack propagation factor.
The tensile strains are calculated (using Boussinesq's equations with Odemark's transformations), for each season and each wheel, at each point (or, more correctly, 300 mm long piece of road), and from this the increase in damage (i.e. reduction of modulus) at this point is determined There is a lower level of the modulus corresponding to a totally cracked material. If the moduli of the bound layers are reduced below a certain value, the modulus of the unbound layers below may also be reduced, due to ingress of moisture.

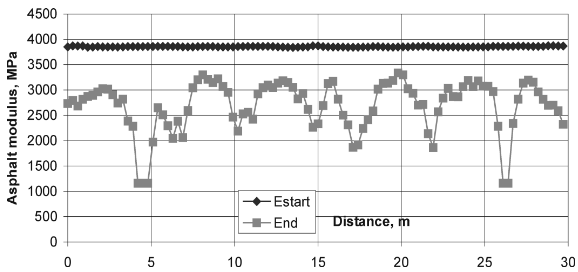

Fig. 10
Longitudinal variation of asphalt modulus, at start and end of the simulation.

5.5 Permanent Deformation

The permanent deformation may also be described by three phases, one of decreasing strain rate, a second of constant strain rate and a third of increasing strain rate. For phase one MMOPP makes use of the following equation:

$$\varepsilon_p = A \times \left(\frac{N}{10^6}\right)^B \times \left(\frac{\sigma_z}{p}\right)^C$$

where ε_p is the plastic strain,
N is the number of load applications,
σ_z is the vertical stress,
p is atmospheric pressure and
A, B and C are constants (A is a function of the modulus).

MMOPP may also include phase two, of constant strain rate (i.e. B=1) if the accumulated permanent strain exceeds a critical value.

It is not necessary to calculate the permanent strains at different depths, the permanent deformation may be calculated directly by using Odemark's transformation with the elastic layer moduli and Boussinesq's equations with the "plastic" layer moduli (i.e. stress over plastic strain) (Ullidtz, 1998).

The plastic deformation of each pavement layer is calculated for each point, under each load for each season. From this the longitudinal profile and the average permanent deformation (rut depth) can be obtained.

5.6 Results

The next three figures show some of the output from a simulation of a pavement section, in terms of roughness (IRI), permanent deformation and relative asphalt modulus, i.e. the asphalt modulus at any point in time divided by the mean modulus of intact material. The simulation was repeated 10 times.

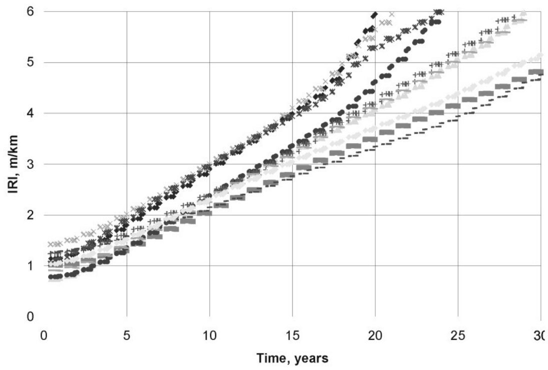

Fig. 11. Roughness as a function of time

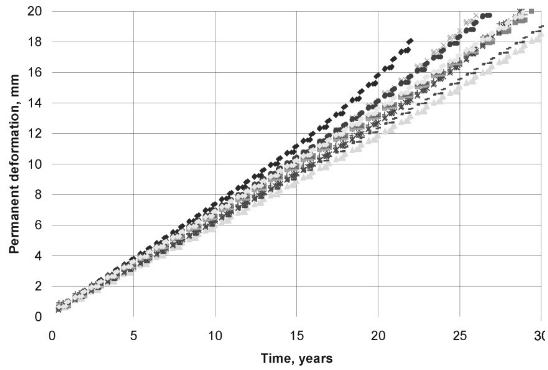

Fig. 12 Permanent deformation as a function of time

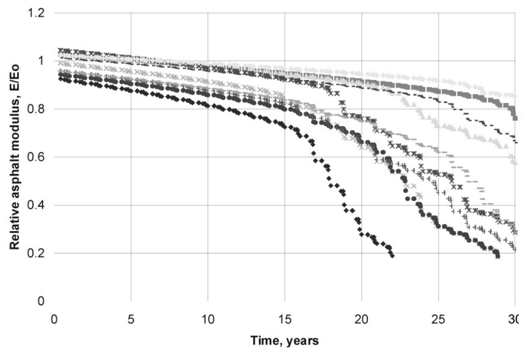

Fig. 13
Relative asphalt modulus as a function of time

Roughness is mostly important for the user comfort and for Vehicle Operating Costs. Permanent deformation will delay the runoff of water and constitutes an accident risk, whereas cracking (relative decrease in asphalt modulus) mostly is of concern to the highway agency, because it will influence the future rate of deterioration. It is, therefore, reasonable to establish limits and required reliability levels for each of these different types of deterioration (it is not reasonable to combine them into an index).

In Table 1 a "failure" limit has been defined for each of the three deteriorations, and the corresponding mean life expectancy in years, as well as the minimum and maximum values of the 10 simulations, have been given. The AASHO Road Test showed that life expectancy follows a logarithmic normal distribution. Therefore the standard deviation has been calculated from the logarithm of the life, and the standard deviation factor (sdf) is 10 raised to the standard deviation.

Table 1
Life expectancy with different reliability levels and different criteria

	IRI	Permanent deformation	Relative modulus
Failure	4.0 m/km	15 mm	0.65
Mean life	19.1	22.3	26.6
Minimum	15.4	19.4	16.9
Maximum	26.7	25.0	> 40
Sdf	1.20	1.08	1.41
85%	15.8	20.6	18.6
99%	12.5	18.6	12.0

With a required reliability level of 85% the roughness criterion will result in the shortest life expectancy, whereas the relative modulus is controlling at a level of 99%.

6. Computer Simulation of a Granular Material

In simulating the performance of a pavement section, a large number of assumptions were made. One of these was that the theories of solid (or continuum) mechanics could be used with pavement materials. Most pavement materials are not solid, however, but consist of grains, and in granular materials, including asphalt, there is no strain field (except within the individual grains, and this has little influence on the overall deformation). There are forces (normal and shear) between the grains, and displacements (translation and rotation) of the grains.

Another assumption was that the deterioration was a function of some response parameter, tensile strain at the bottom of the asphalt layer or compressive strain at the top of the unbound layers. These relationships are purely phenomenological and do not describe the mechanism of deterioration.

To get a better understanding of granular materials the Distinct Element Method (DEM) (Cundall, 1978) is well suited. In DEM each grain is free to move. The calculation of forces and displacements is done in short increments of time. At the beginning of an increment all the forces on the grains should be known. At the start of the simulation the forces could be the force of gravity on each grain or externally applied forces.

From the known forces on the elements, the accelerations, velocities and displacements, at the end of the time step, are calculated. New contacts resulting from the change in geometry are then detected, and finally new forces are calculated from the movements at the contacts and the contact properties. Contact detection is time consuming and is usually not done in every time step.

Most models use an explicit integration of the equations of motion (the second central difference method), whereas Dem2D, used in the following (Ullidtz, 1998), makes use of the “Constant Average Acceleration Method” suggested by Ghaboussi et al. (1993). This is an implicit integration of the equations of motion, where the implicit equations are solved iteratively. This ensures compatibility between accelerations and forces at the end of the time increment.

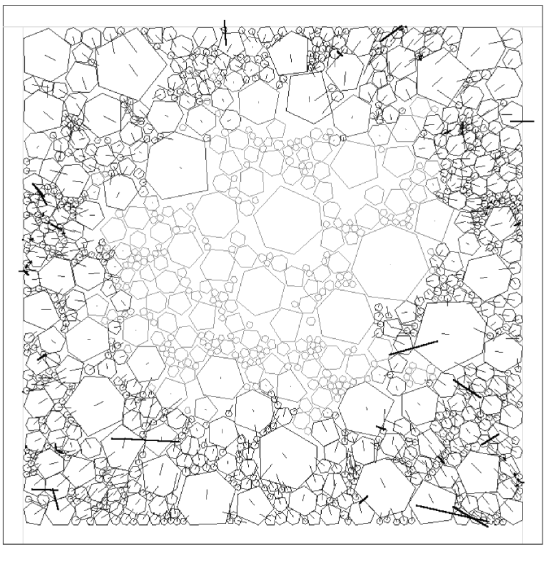

Fig. 14
Sample of grains during compaction

Figure 14 shows a two dimensional (plane strain) sample of “grains” during compaction. In this case a constant stress of 250 kPa was applied to the sides of the box. Once compaction was completed cohesion and cohesive strength were added to the contact points. Cohesion will increase the permissible shear force and the cohesive strength will allow tensile forces at the contact points. The tensile stiffness is also input

Figure 15 shows the same sample, after compaction and addition of cohesion, when the stress in the vertical direction has been reduced to 54 kPa. The sample is, thus, still under compression, and in a continuum the minor principal stress would be a compressive stress of 54 kPa. However, the thick (red) lines indicate tensile forces between the grains, with a maximum tensile force of 7.7% of the tensile strenght.

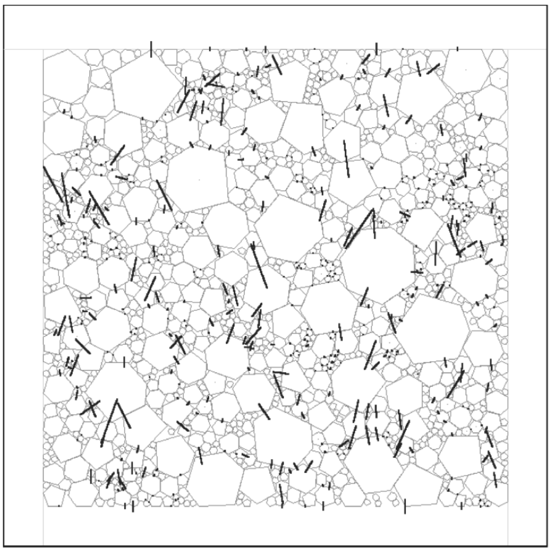

Fig. 15
Sample in compression, but with tensile forces

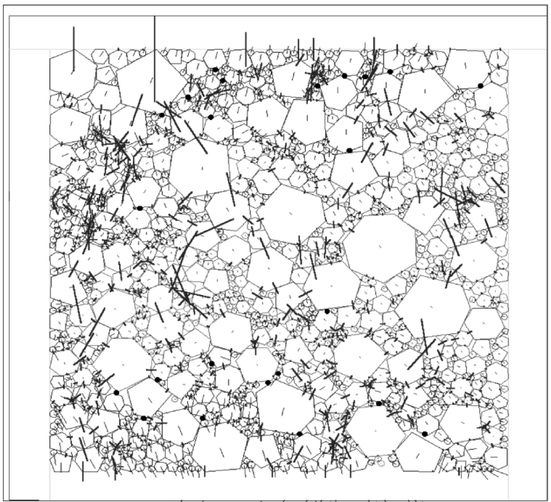

Fig. 16
Partial failure

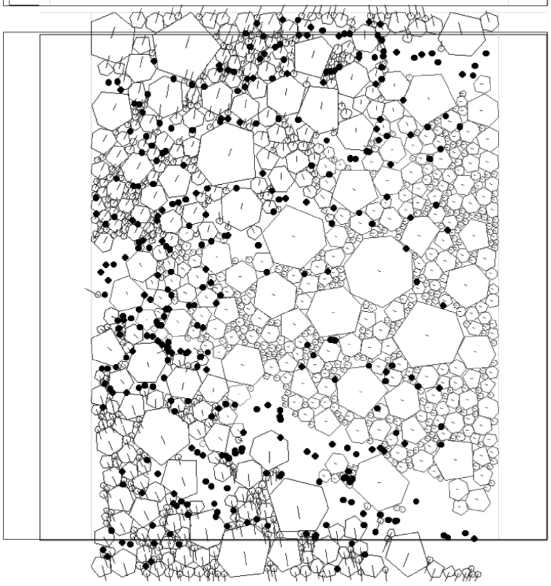

Fig. 17
Complete failure

Figures 16 and 17 show the sample at partial and complete failure. The black points indicate broken contacts. In Figure 16 there are 23 broken contacts.

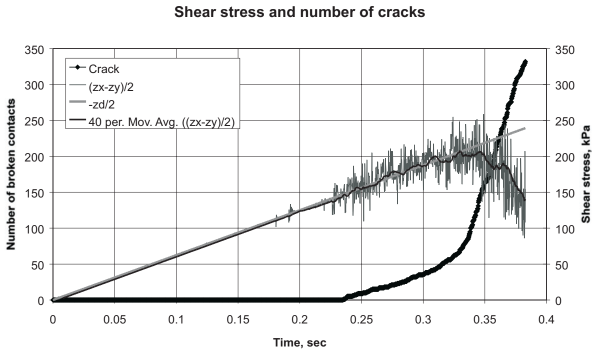

Fig. 18
Shear stress and crack development
as a function of time

Figure 18 shows the number of broken contacts (to the left) and the shear stress (to the right) as a function of the time of loading. With 23 broken contacts (Figure 16) the maximum shear stress has not yet been reached.

One of the advantages of the "virtual" tests compared to real tests, is that tests to failure can be repeated on exactly identical samples. For one sample seven different stress paths were used, as indicated by the thick, black lines in Figure 19.

The stress paths were (for failures from left to right in Figure 19):

1) Pure tension, zx = zy, Δzx = -1.25 MPa/sec, shear stress t = 0
2) Uniaxial tension, zx = 0, Δzy = -1.25 MPa/sec, hydrostatic stress p = t
3) Pure shear, zx = -zy, Δzy = -1.25 MPa/sec, p = 0
4) Uniaxial tension, zx = 250 kPa, Δzy = -1.25 MPa/sec, p = 250 kPa - t
5) Uniaxial compression, zx = 0, Δzy = +1.25 MPa/sec, p = t
6) Pure shear, zx = 250 kPa – zy, Δzy = +1.25 MPa/sec, p = 250 kPa
7) Uniaxial compression, zx = 250 kPa, Δzy = +1.25 MPa/sec, p = 250 kPa + t

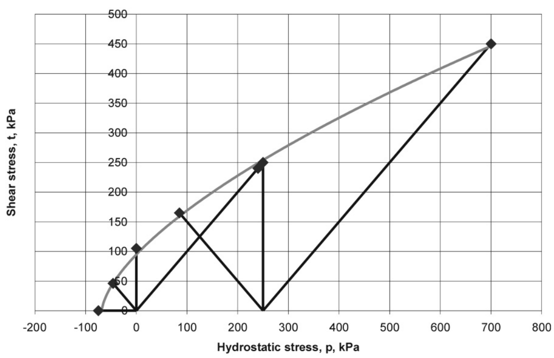

Fig. 19
Different stress paths to failure

The curve fitted to the failure points has the equation:

$$(2t_p/To)^a = (k^a-1)/(k+1) * (2p/To + (k^a+k)/(k^a-1))$$

where t_p could be considered as the permissible shear stress
To is the uniaxial tensile strength (To = 97 kPa)
k is the ratio of uniaxial compressive strength to uniaxial tensile strength (To),
a is a power. a, To and k are determined by minimizing the differences with the measured values, in this case a = 1.53 and k = 5.29

In order to investigate the influence of repeated loading on permanent deformation and failure, a sample was loaded with a constant stress and with a repeated stress (compressive), as shown in Figure 20. The horizontal (or confining) stress was kept at zero.

Both the constant stress and the repeated stress were increased linearly from zero to 562.5 kPa during the first 10 msec, then kept constant at 562.5 kPa for the constant stress loading. For the repeated loading the stress was kept constant for the next 10 msec, decreased to zero during the following 10 msec and then kept constant at zero for 10 msec. This cycle was then repeated.

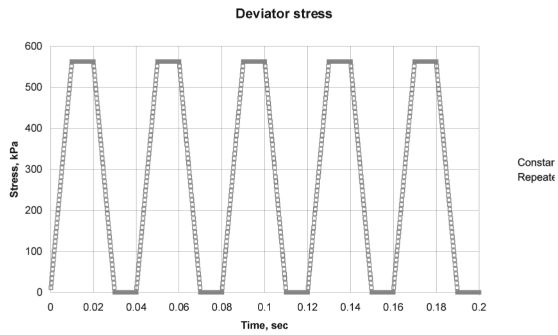

Fig. 20
Constant and repeated deviator stress

Figure 21 shows the deviator strain and the number of broken contacts as a function of time. The deviator strain is shown, in mstrain, at the left ordinate and the number of broken contacts at the right. During the first three loading cycles, or 120 msec of constant load, the (maximum) deviator strain is the same for constant and repeated loading. The repeated loading reveals that about half of the deviator strain is a permanent or plastic strain, and that the permanent strain is increasing slightly with each loading cycle.

After 120 msec there is an increase in the deviator strain under the constant load, at 160 msec four contacts fail and at 235 msec another two contacts fail. For the repeated loading six contacts fail during the sixth loading cycle, at 170 msec. With both types of loading there is, thus, the same number of failures at the beginning of the test, at approximately the same time, and the resulting increase in (maximum) deviator strain is about the same.

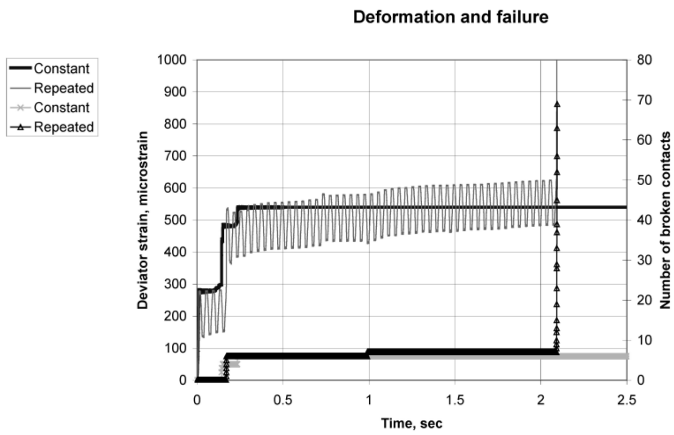

Fig. 21
Different performance under constant and repeated loading

After the initial failures, however, the sample behaves quite differently under the two different types of loading. Under constant loading the deviator strain reaches a level of 540 mstrain and remains stable at this level, with no more movement of the grains. Under repeated loading the permanent part of the deviator strain increases slightly for each loading cycle, whereas the resilient (or reversible) part remains fairly constant. After 25 cycles (about 1 sec) one more failure occurs and after 52 cycles the sample fails completely.

The Distinct Element Method may also be used with larger samples of particles. Figure 22 shows a 1000´2000 mm "box" filled with 3662 particles. The particles have been compacted in two layers to a thickness of 830 mm for the lower layer and 100 mm for the upper layer. Particle size distribution and angularities are different for the two layers. In the upper layer cohesion is assumed between the particles, as well as a permissible tensile force (of 20 N at each contact point). The vectors from the centres of the particles show the displacement during the first 8 msec of loading on a 150 mm plate at the surface of the sample. At this load all contacts were intact.

The displacement field is quite different from what would have been obtained in an elastic solid. An example from a Finite Element calculation, with the same proportions and the same centre line deflection, is shown in Figure 23.

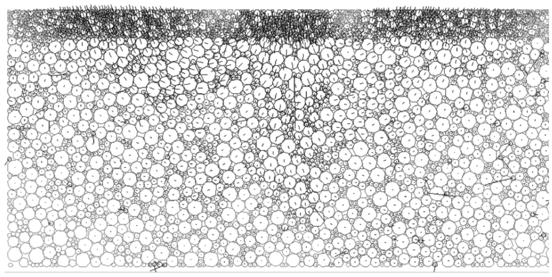

Fig. 22
Displacement field in particulate sample

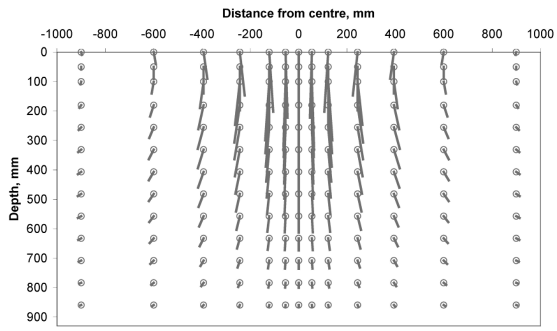

Fig. 23
Displacement field in elastic solid (FEM)

After 27 msec of loading 15 contact points had failed. The position of the failed contact points and the sequence of the first 5 failures are shown in Figure 24.

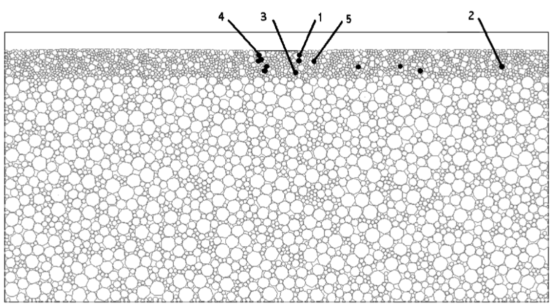

Fig. 24
Location of failed contacts and sequence of failures (first 5 points)

Cracking appears to be neither "bottom up" nor "top down".

7. Conclusion

Since the first International Conference on Structural Design of Asphalt Pavements in Ann Arbor in 1962, the theory of elasticity has been widely used to model pavement structures, and today it forms part of many national pavement design standards. It is still an open question, however, how well the theory of elasticity predicts the pavement response under load. Pavement materials are rarely elastic solids, more often they are particulate, and measurements have repeatedly shown important differences from theoretical values.